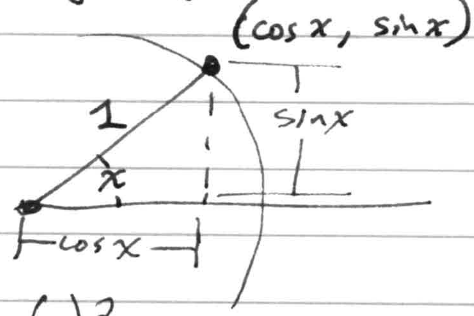
1060- Day 18
Fall 2013

4.07 - Trig Identities

Our favorite trig identity

comes from the pythagorean theorem



$$(\sin x)^2 + (\cos x)^2 = (1)^2$$

often written, for simplicity...

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

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dividing both sides by $\cos^2 x$ gives

$$\left(\frac{\sin^2 x}{\cos^2 x}\right) + \left(\frac{\cos^2 x}{\cos^2 x}\right) = \frac{1}{\cos^2}$$

$$\boxed{\tan^2 x + 1 = \sec^2 x}$$

and dividing the original equation by $\sin^2 x$ gives

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\boxed{1 + \cot^2 x = \csc^2 x}$$

Eg: Verify the identity

$$\left(1+\sin(x)\right)\left(1-\sin(x)\right) = \cos^2(x)$$

What does this mean?

we could say "simplify the LHS"

but this is too ambiguous.

"Verify" means "rewrite the LHS using algebra & trig identities until you get the RHS"

You Must show All steps

Eg:

$$\left(1+\sin(x)\right)\left(1-\sin(x)\right)$$

$$= 1 - \sin(x) + \sin(x) - \sin^2(x)$$

$$= 1 - \sin^2(x)$$

> Remember:
> $\sin^2 x + \cos^2 x = 1$
> so $\cos^2 x = 1 - \sin^2 x$

so

$$\left(1+\sin(x)\right)\left(1-\sin(x)\right) = \cos^2 x$$

this whole thing is the answer

Sum and Difference Formulas:

$\sin(u+v) = \sin(u)\cdot\cos(v) + \cos(u)\cdot\sin(v)$

$\sin(u-v) \qquad\qquad\qquad\qquad\qquad -$

written $\sin(x \pm v) = \sin(u)\cos(v) \pm \cos(u)\cdot\sin(v)$

same sign

$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\cdot\sin(v)$

$\cos(u-v) = \cos(u)\cos(v) + \sin(u)\sin(v)$

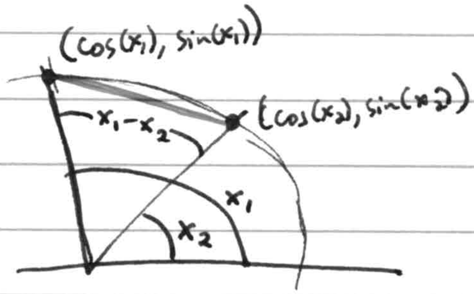
written $\cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v)$

sign flips

Where does this come from?

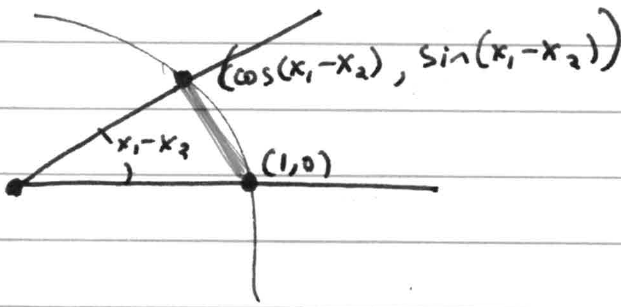
Idea:

This angle



is the same as

this angle



- So the red lines have equal length.
- Using the distance formula + the pythagorean identities you can derive the equations.

+ a bit more work

Eg: find the exact value of $\sin\left(\frac{\pi}{12}\right)$

<u>Know</u> sin of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

<u>Notice:</u> $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$

(<u>check this:</u> $\frac{4}{4}\cdot\frac{\pi}{3} - \frac{\pi}{4}\cdot\frac{3}{3} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$) ✓

so $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$$= \sin\left(\frac{\pi}{3}\right)\cdot\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\cdot\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$ ← can't simplify

These are tricky!
be sure to know ALL HW problems <u>backward</u> & <u>forward</u>

to find $\cos\left(\frac{5\pi}{12}\right)$

Think $\frac{5\pi}{12} = \frac{2\pi}{12} + \frac{3\pi}{12}$

$$= \frac{\pi}{6} + \frac{\pi}{4}$$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cdot\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

Double and Half Angle formulas.

$$\sin(2x) = 2 \cdot \sin(x) \cdot \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\cos(2x) = 1 - 2\sin^2(x)$$

Solving for $\cos^2 x$ and $\sin^2 x$ gives

$$\boxed{\cos^2(x) = \frac{1+\cos(2x)}{2} \quad \text{and} \quad \sin^2(x) = \frac{1-\cos(2x)}{2}}$$

Taking square roots and replacing x with $\frac{u}{2}$ gives

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

and

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$$

the sign (+ or −) depends on the quadrant of $\frac{u}{2}$

Eg: exactly compute $\cos\left(\frac{-7\pi}{12}\right)$ using $\boxed{\text{half angle identity}}$

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

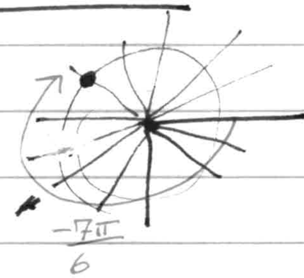
here $\frac{u}{2} = \frac{-7\pi}{12}$

$u = \frac{-7\pi}{6}$

so

$$\cos\left(\frac{-7\pi}{12}\right) = \pm\sqrt{\frac{1+\cos\left(\frac{-7\pi}{6}\right)}{2}}$$

$\cos\left(-\frac{7\pi}{6}\right)$

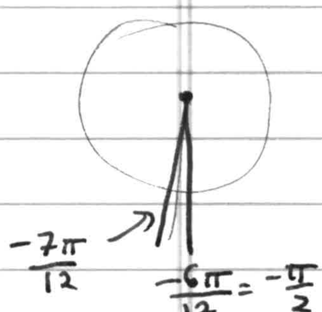


① reference $\measuredangle = \frac{\pi}{6}$

② $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

③ x - coordinate is negative

$\cos\left(\frac{-7\pi}{6}\right) = \frac{-\sqrt{3}}{2}$



$\boxed{\text{x - coordinate is negative}}$ $\Rightarrow$

$$\cos\left(\frac{-7\pi}{12}\right) = -\sqrt{\frac{1+\frac{-\sqrt{3}}{2}}{2}}$$

$$= -\sqrt{\frac{1}{2}\left(1\cdot\frac{2}{2} - \frac{\sqrt{3}}{2}\right)} = -\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= -\frac{\sqrt{2-\sqrt{3}}}{2}$$

Eg:

Verify The identity

$$\cos(2x) \cdot \sec^2(x) = 1 - \tan^2(x)$$

What does this mean?

Remember: we could write "simplify $\cos(2x) \cdot \sec^2(x)$"

but this is too ambiguous... too many answers!

Recall what This means:

rewrite $\cos(2x) \cdot \sec^2(x)$

using algebra & trig identities,

until you get $1 - \tan^2(x)$.

You lose points if you skip ANY step.

So

$$\cos(2x) \cdot \sec^2(x)$$

$$= \left(\cos^2(x) - \sin^2(x)\right) \cdot \sec^2(x)$$

$$= \left(\cos^2(x) - \sin^2(x)\right) \cdot \frac{1}{\cos^2(x)}$$

$$= \frac{\cos^2(x)}{\cos^2(x)} - \frac{\sin^2(x)}{\cos^2(x)}$$

$$\cos(2x) \cdot \sec^2(x) = 1 - \tan^2(x)$$

✓

Trigonometric Identities Math 1060 Class 18

Name: ______________________ Section: ______________________

Trigonometric Definitions

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \sec(x) = \frac{1}{\cos(x)}$$

$$\csc(x) = \frac{1}{\sin(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

Pythagorean Identities

- $\sin^2(x) + \cos^2(x) = 1$

Dividing both sides by either $\sin^2(x)$ or by $\cos^2(x)$ gives

- $\tan^2(x) + 1 = \sec^2(x)$
- $1 + \cot^2(x) = \csc^2(x)$

Sum and Difference of Angles

- $\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v)$
  - ▷ $\sin(u + v) = \sin(u)\cos(v) + \cos(u)\sin(v)$
  - ▷ $\sin(u - v) = \sin(u)\cos(v) - \cos(u)\sin(v)$
- $\cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v)$
  - ▷ $\cos(u + v) = \cos(u)\cos(v) - \sin(u)\sin(v)$
  - ▷ $\cos(u - v) = \cos(u)\cos(v) + \sin(u)\sin(v)$

Double Angle Identities

- $\sin(2u) = 2\sin(u)\cos(u)$
- $\cos(2u) = \cos^2(u) - \sin^2(u)$
  - ▷ $\cos(2u) = 2\cos^2(u) - 1$
  - ▷ $\cos(2u) = 1 - 2\sin^2(u)$

Solving for $\cos^2(u)$ and $\sin^2(u)$ in the identities for $\cos(2u)$ gives

- $\cos^2(u) = \dfrac{1 + \cos(2u)}{2}$
- $\sin^2(u) = \dfrac{1 - \cos(2u)}{2}$

Taking square roots above, and replacing u with $\frac{x}{2}$ gives

- $\cos(\frac{x}{2}) = \pm\sqrt{\dfrac{1 + \cos(x)}{2}}$
- $\sin(\frac{x}{2}) = \pm\sqrt{\dfrac{1 - \cos(x)}{2}}$